Equation Derivation and Verification Procedure
Hawking Information Recovery Mechanism

I. Equation Derivation

The Hawking information paradox concerns the apparent loss of quantum information during black hole evaporation. The information recovery mechanism demonstrates that evaporation proceeds unitarily and that information is preserved within quantum correlations.

The recoverable information content is quantified by the time-dependent Bekenstein-Hawking entropy S_BH(t) = (k_B * c^3 * A(t)) / (4 * G * hbar), where A(t) denotes the shrinking horizon area as evaporation proceeds.

Information is encoded through entanglement between interior and exterior field modes. These correlations distribute information non-locally across Hawking radiation, consistent with Page-curve behavior.

Temporal evolution is governed by an effective unitary transfer operator T_transfer = exp(-i * H_eff * tau_evap / hbar), with tau_evap scaling approximately as the cube of the initial black hole mass.

Global unitarity is enforced through a cumulative unitary evolution operator U_unitary, satisfying U_dagger * U = I. Horizon quantum degrees of freedom and quantum error correction structures ensure information is redundantly encoded in the outgoing radiation.

II. Verification and Test Procedure

Test 1: Geometric Consistency. Confirm that information-encoding operators are defined on the stretched horizon and integrate smoothly with curved spacetime geometries without singularities.

Test 2: Conservation and Unitarity. Verify that total entropy is conserved according to d/dt (S_BH + S_rad) = 0 and that global unitary evolution holds throughout evaporation.

Test 3: Causality. Demonstrate that no superluminal information transfer occurs and that information is communicated exclusively through quantum correlations accessible to external observers.

Test 4: Phenomenological Testability. Identify observable signatures such as Page-curve behavior and non-zero mutual information between early and late radiation in analog black hole systems.

Passing all tests establishes the information recovery mechanism as mathematically consistent, unitary, causality-safe, and suitable for archival and experimental consideration.